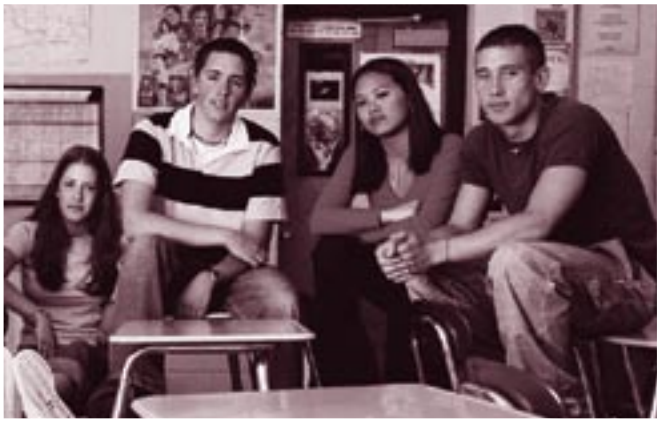
High School Content Expectations



MATHEMATICS

- Quantitative Literacy
- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability

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**Grade Level Content Expectations Committee*

Welcome to Michigan's High School Mathematics Content Standards and Expectations

Why Develop Content Standards and Expectations for High School?

In 2004, the Michigan Department of Education embraced the challenge to initiate a "high school redesign" project. Since then, the national call to create more rigorous learning for high school students; the Cherry Commission Report highlighting several goals for Michigan, including the development of high school content expectations that reflect both a rigorous and a relevant curricular focus; and the implementation of the Michigan Merit Exam, which must be based on rigorous high school learning standards; have led the Michigan Department of Education's Office of School Improvement to spearhead the development of high school content expectations for English Language Arts and Mathematics. Each content area work group was chaired by a nationally known scholar in the respective field. In each area, a small work group of academicians has conducted a scholarly review and has identified content standards and expectations.

An Overview

The expectations contained in this document reflect best practices and current research in the teaching and learning of mathematics. They build from the *Michigan Mathematics Curriculum Framework Standards and Benchmarks* (1996), the *Career and Employability Skills Content Standards and Benchmarks (2001)*, and extend the *Michigan K-8 Mathematics Grade Level Content Expectations* (2004) as appropriate for grades 9-12.

The standards and expectations are closely aligned with national standards as described in ACT's *Standards for Transition*®, American Diploma Project's *Ready or Not: Creating a High School Diploma That Counts* (2004), the National Council of Teachers of Mathematics *Principles and Standards for School Mathematics (2000)*, and the National Assessment Governing Board's *Mathematics Framework for the 2003 National Assessment of Educational Progress (NAEP)*. Students whose work is guided by these standards and expectations will be prepared both for college and for the workplace.

Understanding the Organizational Structure

The expectations in this document are divided into four strands with multiple standards within each, as shown below. The skills and content addressed in these standards will, in practice, be woven together into a coherent, integrated Mathematics curriculum. The standards are comprehensive and are meant to be used as a guide to curriculum development.

STRAND 1 Quantitative Literacy (L)	STRAND 2 Algebra & Functions (A)	STRAND 3 Geometry & Trigonometry (G)	STRAND 4 Statistics & Probability (S)
STANDARDS (and number of expectations for each standard)			
L1: Representations and Properties of Number and Other Systems (10)	**A1: Symbols, Expressions, and Operations** (8)	**G1: 2-Dimensional Figures and Their Properties** (52)	**S1: Univariate Data** (12)
L2: Calculation, Algorithms, and Estimation (19)	**A2: Functions** (55)	**G2: 3-Dimensional Figures and Their Properties** (10)	**S2: Bivariate Data** (11)
L3: Measurement and Precision (6)	**A3: Equations & Inequalities** (16)	**G3: Relations Between Figures** (16)	**S3: Surveys & Experiments** (9)
	A4: Mathematical Modeling (13)	**G4: Transformations of Figures in the Plane** (19)	**S4: Statistical Inference** (13)
		G5: Trigonometry and Vectors (19)	**S5: Reasoning** (8)
			S6: Simulation and the Law of Large Numbers (3)
			S7: Probability Models and Calculating Probabilities (8)
			S8: Probability Distributions (4)

Curriculum and Assessment

This document is intended to support conversations at the school and district level that result in rigorous and relevant curriculum that incorporates these content standards.

As stakeholders (e.g., teachers, administrators, school board members, parents, community members, students, local legislative representatives) work with these standards, they should consider the following questions:

- How are these content standards and expectations reflected in our curriculum and instruction already?
- Where do we need to strengthen our curriculum and instruction to more fully realize the intent of these standards and expectations?
- What opportunities do these standards and expectations present to develop new and strengthen existing curriculum, leading to instructional excellence?
- How do we implement these standards and expectations taking into account what we know about our students, school, and community?
- How will we assess the effectiveness with which our students and schools are meeting these standards and content expectations?
- How can we use school-based assessments (e.g., student portfolios, school-based writing assessments, teacher or classroom research, district-level assessments) to make data-driven decisions about teaching and learning?

Through conversations about questions such as these, and building upon the multitude of existing strengths in our current high schools, voices of all stakeholders will participate in the important and continuing process of shaping instructional excellence in Michigan schools and preparing students in Michigan schools for college and the workplace.

Mathematics

Mathematical understandings and skills are essential elements for meaningful participation in the global information society. US expectations in mathematics for high school students have not kept pace with expectations in high-achieving countries around the world. And, expectations about who can do mathematics in the US have led to inequitable and unacceptably low opportunities to learn for students living in poor and urban communities. In Michigan, the K-8 Mathematics Grade Level Expectations represent a major step forward in raising expectations in mathematics for all students; these high school expectations assume the ambitious foundation of the K-8 GLCEs and are intended to equip all students with a solid background for continued postsecondary study in any area, as well as with skills and knowledge essential for the workplace.

The high school mathematics standards are organized in four interconnected strands: Quantitative Literacy, Algebra and Functions, Geometry and Trigonometry, and Statistics and Probability. The standards include a strong emphasis on problem solving and mathematical reasoning. Elements of discrete mathematics are incorporated into these four strands. Technological advances affect what is possible to learn, and what is necessary to learn, in high school mathematics; these standards reflect this change through the inclusion of particular topics, and reference to the use of specific electronic tools. We see these four strands as fundamentally interconnected and also arranged to reflect the sequencing and emphases in the mathematical ideas that are central to high school.

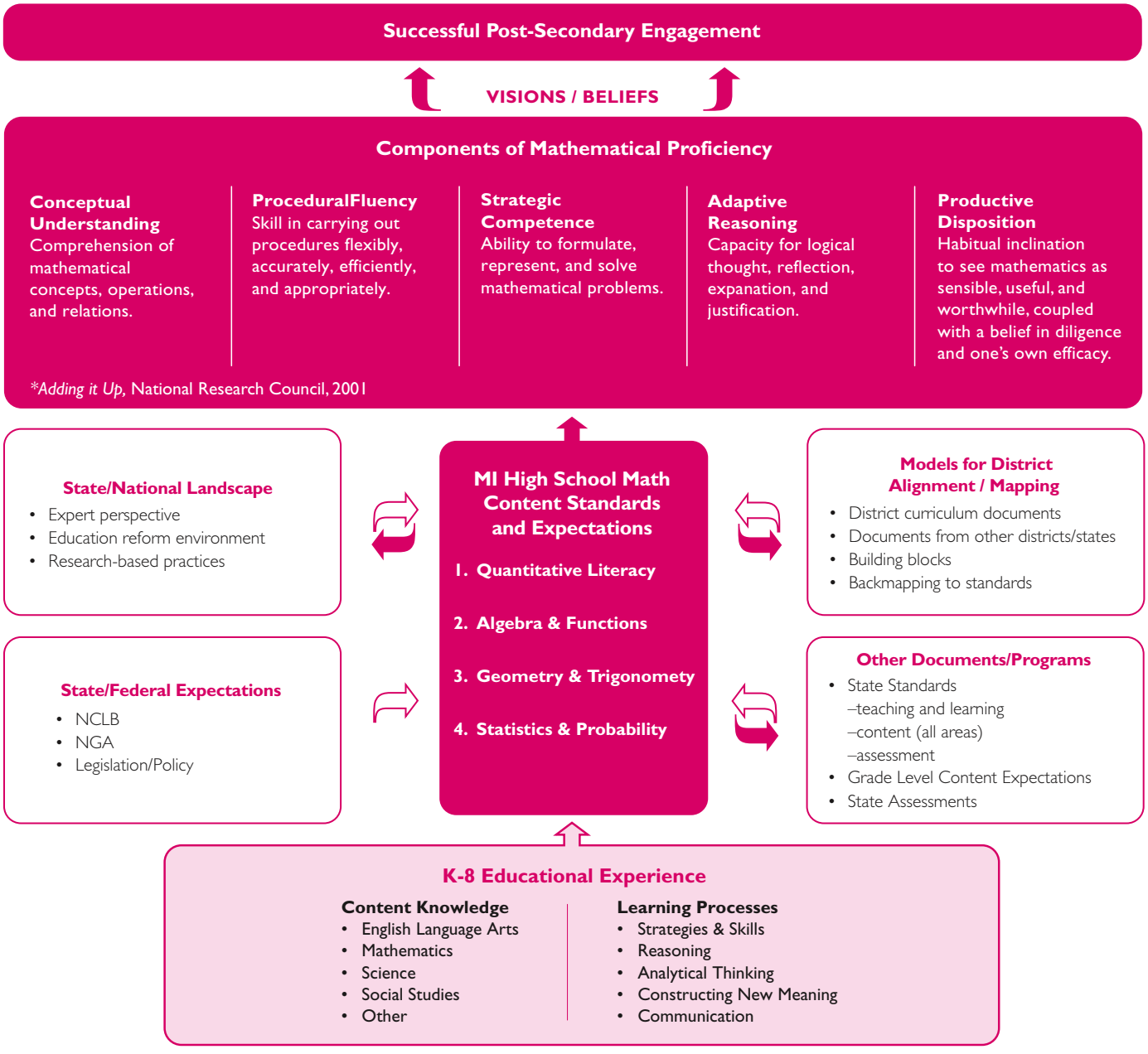
The following principles underlie the standards and expectations:

- It is essential to hold high expectations in mathematics for all students for completion of high school, whether they will enter the workforce or go on to postsecondary education.
- The K-8 Mathematics GLCE are ambitious in their expectations, and these high school expectations assume them as a foundation and build on them.
- The focus is on the expectations, across four strands, which are not yet mapped into course arrangements. Such mapping, whether to "traditional" course titles like Algebra I, Geometry, or Algebra II, or into courses that integrate the material, is a complex process.
- The topics within each strand have been arranged to show mathematical growth and to illustrate mathematical trajectories of ideas that build on one another, when possible.

Sometimes we have flagged expectations as "advanced". These indicate expectations that are beyond what is expected of all high school graduates. We have not created a comprehensive listing of all of the advanced objectives that would be part of a four-year high school program. We believe that all students should take mathematics in their fourth year of high school to ensure a smooth transition to their next set of experiences.

Preparing Students for Successful Post-Secondary Engagement

As we use these standards and expectations to develop rigorous and relevant units of instruction, powerful and engaging learning activities, and challenging high school curricula, it is critical to keep in mind that content knowledge alone will not provide adequate preparation for success in entry-level university courses or entry-level positions in today's workforce. Students must be able to apply knowledge in new situations, to solve problems by generating new ideas, to make connections between what they read and hear in class and the world around them and to the future, and to develop leadership qualities while still in high school. As educators, we must model for and develop in students the cognitive skills, habits of mind, or dispositions that will result in mathematical proficiency and successful post-secondary engagement.



STRAND 1: QUANTITATIVE LITERACY (L)

"In an increasingly complex world, adults are challenged to apply sophisticated quantitative knowledge and reasoning in their professional and personal lives. The technological demands of the workplace, the abundance of data in the political and public policy context, and the array of information involved in making personal and family decisions of all types necessitate an unprecedented facility not only with fundamental mathematical, statistical, and computing ideas and processes, but with higher-order abilities to apply and integrate those ideas and processes in a range of areas."[1]

Traditionally, for many U.S. students, a significant component of their high school mathematics education has included a revisiting of the processes of arithmetic. The Michigan Grade Level Content Expectations in Mathematics for grades K-8 prescribe a thorough treatment of number, including strong emphasis on computational fluency and understanding of number concepts, to be completed largely by the sixth grade. The expectations in this strand assume this strong foundation in number and build on it, as well as expand it, into a definition of secondary school quantitative literacy for all students. In particular, these expectations assume fluency (that is, efficiency and accuracy) in calculation with the basic number operations involving rational numbers in all forms (including percentages and decimals), without calculators.

The K-8 expectations are reinforced and applied in more complex situations through these Quantitative Literacy expectations, particularly through connections and applications of number ideas to other areas of mathematics, such as algebra, geometry, and statistics. Number representations and properties extend from the rational numbers into the real and complex numbers, as well as other systems that students will encounter both in the workplace and in more advanced mathematics. The expectations for calculation, algorithms and estimation provide experience with important uses of number in a range of real-life contexts that students are likely to encounter. Measurement and precision is an area of number that ties closely to geometry; finally, a look at number systems from a more comprehensive standpoint is designed to ensure that students can make the important connections between algebra and arithmetic.

These Quantitative Literacy expectations also include focus on the kinds of numerical and quantitative arguments that arise on a daily basis for citizens in a technological, global society, and will ensure that the high school students of Michigan are adept in such modes of reasoning.

[1] Estry, D., & Ferrini-Mundy, J. (January, 2005). Quantitative Literacy Task Force Final Report and Recommendations. East Lansing: Michigan State University.

STANDARD L1: REPRESENTATIONS AND PROPERTIES OF NUMBER AND OTHER SYSTEMS

Students represent and order numbers, and use the properties of special numbers.

L1.1 Representations and Relationships

L1.1.1 Represent numbers in scientific notation, and interpret calculator or computer displays of numbers given in scientific notation.

L1.1.2 Represent absolute value relationships, both abstract and applied (e.g. tolerances) on the number line, as intervals and points.

L1.1.3 Know how to convert terminating and repeating decimals into rational number forms, and recognize that the decimal expansion of a rational number terminates if and only if its denominator in reduced form has only 2's and 5's as factors.

L1.1.4 Represent, interpret and compare large and small numbers (e.g., chance of winning a lottery, national debt, distances in space, size of atomic particles).

L1.1.5 Recognize that the complex number i is the solution to $x^2 = -1$, and represent complex numbers on the Cartesian plane, in $a + bi$ form, or as $re^{i\theta}$. (ADV)

L1.1.6 Represent vectors as directed line segments on the Cartesian plane.

L1.1.7 Represent a 2 x 2 system of linear equations as a matrix equation.

L1.2 Understanding Properties of Special Numbers and Matrices

L1.2.1 Know that multiplication or division of a number by 1 (in any form) yields the original number; know that the multiplication of a 2 x 2 matrix by the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ yields the original matrix.

L1.2.2 Explain and apply special properties of zero.

a. addition of 0 (in any form) to a number, and subtraction of 0 from a number, gives the original number

b. multiplication of a number by 0 gives 0

c. the Zero Product Principle (if the product of two numbers is 0, then at least one of the numbers is 0)

d. division by 0 and 0^0 are undefined

e. $a^0 = 1$ for any non-zero number a

f. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

L.1.2.3 Understand the significance of the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ (because of their role in basic right triangle trigonometry), π (because of its relationships with circles), and e (because of its role in continuously compounded interest).

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

Students calculate fluently, estimate proficiently, and describe and use algorithms.

L2.1 Calculation with Real and Complex Numbers, Vectors, and Matrices

L2.1.1 Compute with square roots of whole numbers (e.g., $\sqrt{3}$ and $5\sqrt{3}$), π, and e, and approximate these and other irrational numbers as rational numbers for computation.

L2.1.2 Compare the different effects of multiplying, dividing, or exponentiating a real number by a number less than 0, a number between 0 and 1, and a number greater than 1.

L2.1.3 Find inverses of complex numbers and 2 x 2 matrices.

L2.1.4 Evaluate numerical expressions of real numbers that include fractional and negative exponents, and compute with numbers in scientific notation using properties of exponents.

L2.1.5 Understand the relationships between a number and its logarithm (base 10), and find logarithms (base 10) of numbers expressed in scientific notation.

L2.1.6 Calculate (e.g., add, multiply, raise to integer powers) with complex numbers.

L2.1.7 Add two vectors or matrices; multiply a vector or a matrix by a scalar; multiply two 2 x 2 matrices; and interpret in applied settings.

L2.1.8 Solve equations involving 2 x 2 matrices and vectors, and interpret in applied settings.

L2.2 Explaining, Writing, and Using Algorithms and Estimation Procedures

L2.2.1 Find the nth term in an arithmetic, geometric, or recursively described sequence; use summation notation and compute sums of finite arithmetic and geometric sequences of numbers.

L2.2.2 Approximate square roots with an iterative algorithm.

L2.2.3 Apply algorithms for finding weighted averages, composite indices, apportionment procedures, and voting and polling algorithms (e.g., consumer price index, college rankings, GNP, funding formulas, apportionment of US House of Representatives, fair division techniques, and majority, plurality, points-for-preference, and runoff voting techniques).

L2.2.4 Describe, explain, and apply the counting techniques of permutations and combinations, and connect combinations to Pascal's Triangle.

L2.2.5 Determine and compare order of magnitude of estimates of such quantities as: number of people in a hotel, distance measured in city blocks, crowd size, and population of a city, a state, and a country.

L.2.3 Reasoning about Numbers, Systems, and Quantitative Situations

L2.3.1 Describe the changes that occur in axioms when moving from the positive integers to the integers (e.g., closure under subtraction is gained), from the integers to the rational numbers (e.g., all rational numbers have multiplicative inverses), and from the rational numbers to the real numbers.

L2.3.2 Given algebraic expressions or relationships, describe how changes in one variable affect the expression (e.g., given a/b, if b is doubled, how does the expression change?), and how algebraic operations affect the relationships (e.g., given $a < b$, if both sides are multiplied by –2, how does the relationship change?).

L2.3.3 Justify numerical relationships (e.g., show that the sum of even integers is even, or $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ or properties of numbers (e.g., show that $\sqrt{2}$ is irrational; recognize that, except for special cases, the nth roots of integers are irrational).

L2.3.4 Evaluate inductive and deductive arguments, and the use of examples and counterexamples in quantitative arguments (e.g., examining the fallacies of advertising, critiquing the analyses of election results and other public opinion polls, etc.).

L2.3.5 Use terms and concepts from formal logic (e.g., axiom, proposition, negation, truth and falsity, implication, if-and-only-if, converse, inverse, contrapositive) to reason about mathematical concepts and quantitative situations.

L2.3.6 Differentiate between statistical arguments (inferences based on empirical data) and logical arguments (arguments verified using mathematical proof); distinguish between relevant and irrelevant information, and identify missing information in arguments.

STANDARD L3: MEASUREMENT AND PRECISION

Students apply measurement units and calculations.

L3.1 Measurement Units, Calculations, and Scales

L3.1.1 Convert units of measurement between systems for a variety of systems.

L3.1.2 Explain how arithmetic operations affect the dimensions of units, and carry units through calculations so that results include correct units.

L3.1.3 Describe the relationship between logarithmic scales (e.g. Richter scale, pH scale) in comparison to linear scales.

L3.2 Understanding Error; No Measurement is Exact

L3.2.1 Determine what is a reasonable degree of accuracy for measurement in a given situation and express accuracy through use of significant digits (e.g., 3 is not as accurate as 3.0).

L3.2.2 Describe how errors in measurements are magnified by calculation; describe and explain round-off error, rounding, and truncating, and know how to compensate for the inaccuracies it introduces.

L3.2.3 Understand and use concepts of accuracy, error, tolerance, and accumulated error in applied situations (such as in making a pie chart, and the rounding errors that may result in less than 360 degrees).

STRAND 2: ALGEBRA AND FUNCTIONS (A)

In the middle grades, students see the progressive generalization of arithmetic to algebra. They learn symbolic manipulation skills and use them to solve equations. They study simple forms of the building block functions, for example, linear, quadratic, and power functions as represented by tables, graphs, symbols, and verbal descriptions.

In high school, students continue to develop their "symbol sense" by examining the underlying structure of expressions, equations, and functions. They construct a conceptual framework for analyzing any function and, using this framework, they revisit the functions they have studied before in greater depth. By the end of high school, their catalog of functions will encompass linear, exponential, quadratic, power, polynomial, rational, logarithmic, trigonometric, and recursive functions.

Students will view algebra not only as a theoretical tool for analyzing and describing mathematical relationships, but they will also experience the power of algebraic thinking in the context of applications by studying the mathematical modeling of real-world problems.

STANDARD A1: SYMBOLS, EXPRESSIONS, AND OPERATIONS

Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms.

A1.1 Recognition, Construction, and Interpretation of Expressions (linear, exponential, quadratic, power, polynomial, and rational)

A1.1.1 Give a verbal description of an expression that is presented in symbolic form, and write an algebraic expression from a verbal description.

A1.1.2 Evaluate a variable expression.

A1.2 Transformation of Expressions Into Equivalent Forms

A1.2.1 Know and apply the definitions and properties of exponents, roots, and fractions in algebraic expressions.

A1.2.2 Use the distributive property to expand expressions.

A1.2.3 Factor expressions using greatest common factor, grouping, and trial and error methods.

A1.2.4 Use special product formulas to expand expressions or factor them including:

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)(x - y) = x^2 - y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

A1.2.5 Simplify exponential and logarithmic expressions using the inverse relationship between exponents and logarithms.

A1.2.6 Add, subtract, multiply, divide, and simplify polynomials and rational expressions.

STANDARD A2: FUNCTIONS

Students understand functions and their representations. They perform transformations, combinations, compositions and find inverses. Students classify functions and know the characteristics of each class.

A2.1 Concepts and Representations of Functions

A2.1.1 Read, interpret, and use function notation, and evaluate a function at a value in its domain.

A2.1.2 Analyze, interpret, represent, and translate among relationships expressed in symbols, graphs, tables, diagrams, or words.

A2.1.3 Distinguish between independent and dependent variables.

A2.1.4 Recognize whether a relationship given in symbolic, table, or graphical form is a function.

A2.2 Rates of Change and General Attributes of Functions

A2.2.1 Recognize and interpret patterns of change and distinguish constant rates of change (i.e., slope) from variable rates of change (e.g., increasing at an increasing rate).

A2.2.2 Calculate and interpret net change and rate of change.

A2.2.3 Describe the major similarities and differences between rates of change of different families of functions.

A2.2.4 Identify the units associated with the rate of change in a quantity for a variety of situations.

A2.2.5 Find the domain and range of the function from its formula, graph, or rule, and determine the realistic domain and range of the function in a given context.

A2.2.6 Identify the zeros, intervals of increase and decrease, maximum and minimum values, and end behavior given the symbolic form of a function.

A2.2.7 Determine whether or not the function has an inverse.

A2.2.8 Identify the family to which the function belongs and know the general symbolic forms that can be used to represent the family.

A2.2.9 Recognize and interpret the key features of the graph of a function, including its shape, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, evenness or oddness, rate of change, periodicity, and continuity.

A2.3 Operations, Transformations, and Inverses

A2.3.1 Combine functions by addition, subtraction, multiplication and division, and represent the results graphically and symbolically.

A2.3.2 Write an expression for the composite of two functions, given symbolic forms of two functions.

A2.3.3 Graph transformations of basic functions including vertical and horizontal shifts, stretches and shrinks, as well as reflections across the x- and y-axes.

A2.3.4 Write function rules for reflections across the x-axis or y-axis, translations, or stretches (or composites of these transformations) of basic functions.

A2.3.5 Know and interpret the function notation for inverses.

A2.3.6 Find the inverse of a function if it exists.

A2.3.7 Verify that two functions are inverses using composition.

A2.3.8 Discuss the characteristics of functions and their inverses, including one-to-oneness, domain, and range. (ADV)

A2.4 Families of Functions

A2.4.1 Identify a function as a member of a basic family of functions by examining its symbolic form, table, graph, or verbal description.

A2.4.2 Describe the table and graph patterns expected in various types of functions.

A2.4.3 Write the general symbolic forms of the equations that characterize each family of functions.

A2.5 Linear Functions

A2.5.1 Know and interpret the relationship between the coefficients of a linear equation and the slope and x- and y-intercepts of its graph.

A2.5.2 Write, graph, and interpret equations of the form $x = \mathrm{h}$ and $y = \mathrm{k}$ and their vertical and horizontal line graphs.

A2.5.3 Write the equation for a line given the slope and a point on the line, or two points on the line.

A2.5.4 Relate slope to parallelism and perpendicularity and write the equation for a line using properties of parallel and perpendicular lines.

A2.5.5 Rewrite an equation for a line in standard form or slope-intercept form.

A2.6 Exponential and Logarithmic Functions

A2.6.1 Recognize and describe the pattern of change characteristic of exponential functions.

A2.6.2 Describe major similarities and differences between linear and exponential patterns of change.

A2.6.3 Write the symbolic form and sketch the graph of an exponential function given appropriate information.

A2.6.4 Interpret the constant ratio in a geometric sequence as the base of the associated exponential function.

A2.6.5 Know that the base of an exponential function determines increase or decrease and rate of growth or decay.

A2.6.6 Convert between the forms $f(x) = a^x$ and $f(x) = e^x$. (ADV)

A2.6.7 Recognize the inverse relationship between logarithmic and exponential functions.

A2.6.8 Recognize graphs of the form $f(x) = ab^x$, $f(x) = \log_a x$, $f(x) = e^x$, $f(x) = \ln x$.

A2.6.9 Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.

A2.7 Quadratic Functions

A2.7.1 Write the symbolic form and sketch the graph of a quadratic function given appropriate information.

A2.7.2 Recognize the elements of a parabola (vertex, axis of symmetry, direction of opening, sharpness) and their connection to the coefficient(s) of the symbolic form.

A2.7.3 Describe the number of real solutions for quadratic equations and illustrate with graphs.

A2.7.4 Convert between standard and vertex form of parabolas by Completing the Square.

A2.8 Power Functions (including roots, cubics, quartics, etc.)

A2.8.1 Recognize characteristics of direct and inverse variation functions.

A2.8.2 Graph quadratic, cubic, and radical power functions, noting symmetry.

A2.9 Polynomial Functions

A2.9.1 Analyze graphs of polynomial functions of degree higher than 2 and understand the effects of degree, lead coefficient, and multiplicity of real zeros on the graph.

A2.9.2 Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function.

A2.9.3 Find the real zeros of a polynomial function.

A2.10 Rational Functions

A2.10.1 Analyze graphs of rational functions and understand the relationship between the zeros of the numerator and denominator and intercepts, asymptotes, and domain.

A2.11 Piecewise Functions

A2.11.1 Analyze the graphs of piecewise functions (including step and absolute value functions) noting points of discontinuity and intervals over which the function is represented by each expression.

A2.12 Trigonometric Functions

A2.12.1 Know and apply the unit circle definitions of the trigonometric functions (sine, cosine, tangent).

A2.12.2 Know and apply the definition of a radian and convert between degree and radian measures.

A2.12.3 Know the exact sine, cosine, and tangent values for $0, \pi/2, \pi/3, \pi/4, \pi/6$, and multiples of π and use those values to find other trigonometric values.

A2.12.4 Analyze the graphs of the sine, cosine, and tangent functions noting period, amplitude, and location of maxima and minima.

A2.12.5 Graph transformations (changing period, amplitude, phase, and midline) of basic trigonometric functions and understand the relationship between constants in the formula and the transformed graph.

A2.12.6 Define and graph inverse trigonometric functions ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$).

STANDARD A3: EQUATIONS AND INEQUALITIES

Students determine appropriate techniques for solving each type of equation or system of equations, apply the techniques correctly, and draw conclusions from the solutions. They know and apply common formulas.

A3.1 Solutions of Equations and Inequalities

A3.1.1 Write equations and inequalities with one or two variables to represent mathematical or real-life situations.

A3.1.2 Determine whether an equation has 0, 1, several, or infinitely many solutions (including use of discriminant), and decide whether solutions are real numbers or complex numbers.

A3.1.3 Understand and describe the relationships among the solutions of an equation, the zeros of the associated function, and the x-intercepts of its graph.

A3.1.4 Select an appropriate technique to solve (or estimate solutions for) an equation or inequality including graphical, algebraic (e.g. simplification operations, formulas, properties, theorems, or factoring), technological (e.g. calculator, spreadsheets, computer algebra systems [CAS]) and systematic search methods, apply correctly, and distinguish between approximate and exact solutions.

A3.1.5 Decide whether a solution is reasonable in the context of the situation represented.

A3.1.6 Solve linear equations and inequalities in one variable, absolute value equations and inequalities.

A3.1.7 Solve equations involving rational expressions, quadratic equations (real and complex solutions), and equations involving polynomials.

A3.1.8 Solve exponential and logarithmic equations.

A3.1.9 Solve equations including radical expressions.

A3.1.10 Solve trigonometric equations over a variety of domains.

A3.1.11 Solve systems involving up to three linear equations using technology, graphing, substitution, elimination, and matrix methods.

A3.2 Formulas

A3.2.1 Select an appropriate formula to apply in a contextual situation.

A3.2.2 Solve an equation involving several variables for one variable in terms of the others.

A3.2.3 Know and apply common formulas fluently (e.g., slope, distance between two points, midpoint of a segment, Pythagorean Theorem, Quadratic Formula, compound interest, distance = rate · time).

A3.3 Matrix Algebra

A3.3.1 Use adjacency matrices to represent vertex-edge graphs and to indicate relationships among members of a set.

A3.3.2 Use matrix operations to analyze an adjacency matrix, including row sum, column sum, and finding powers of a matrix, and interpret the results.

STANDARD A4: MATHEMATICAL MODELING

Students select a function to model a real-world situation in order to solve applied problems. They analyze iterative processes and know that certain situations are best modeled by recursive functions.

A4.1 Models of Real-world Situations Using Families of Functions

A4.1.1 Identify the family of function best suited to a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; or trigonometric functions for periodic phenomena).

A4.1.2 Adapt the general symbolic form of a function to one that fits the specifications of the situation by using the information given to replace arbitrary constants with numbers.

A4.1.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled.

A4.1.4 Use linear programming to represent and solve real-life problems.

A4.2 Iteration and Recursion

A4.2.1 Explain sequences and series as iterative processes.

A4.2.2 Model various types of growth and decay processes using recursive functions, including population growth and decay, radioactive decay, and compound interest; construct recursive models for such situations, and distinguish between arithmetic and geometric growth.

A4.2.3 Describe the effects of changing various parameters on the long-term behavior of recursion equations in forms such as $A_n = rA_{n-1} + b$, as well as the effects on the situations they model.

A4.2.4 Iterate functions and describe the resulting patterns, especially the long-term behavior.

A4.2.5 Use linear, exponential, and polynomial functions to model discrete situations.

A4.3 Parametric Equations (ADV)

A4.3.1 Evaluate parametric equations for given values of the parameter. (ADV)

A4.3.2 Graph curves represented by parametric equations. (ADV)

A4.3.3 Translate between parametric equations and rectangular equations. (ADV)

A4.3.4 Find parametric equations given a graph of a curve. (ADV)

STRAND 3: GEOMETRY AND TRIGONOMETRY (G)

In Grades K–8, students learn to recognize and draw such figures as triangles, rectangles, circles, rectangular solids, cylinders, and spheres. They also examine similarities and differences among geometric shapes; study relevant properties such as angle measure, area, volume, and symmetry; and use these properties to solve problems related to the real world.

In Grades 9–12, students build upon the geometry studied in earlier grades by solving more complex problems about familiar figures. They also study other 2- and 3-dimensional figures, e.g. vertex-edge graphs and regular polyhedra; relations between geometric figures, e.g. congruence and similarity; transformations of figures, e.g. reflections, glide reflections, and their applications to tessellations of the plane; and trigonometry and vectors. To investigate patterns and to make and test conjectures about geometry and trigonometry in high school, students will use tools such as the compass and straight edge of ancient Greece, as well as modern technology such as graphing calculators or dynamic geometry software. To prove that conjectures are true, students are expected to use various forms of deductive reasoning described in the Quantitative Literacy strand, including both direct and indirect proof.

STANDARD G1: TWO-DIMENSIONAL FIGURES AND THEIR PROPERTIES

Students represent discrete geometric figures, polygons, and conic sections and apply their properties in solving problems and justifying arguments. Advanced students use polar coordinates and parametric equations and their properties.

G1.1 Discrete Geometric Figures

G1.1.1 Construct and use vertex-edge graphs to model and solve problems involving critical path analysis, minimal spanning trees, shortest routes, and Euler paths and circuits.

G1.1.2 Construct and use vertex-edge graphs to model and solve problems involving Hamiltonian paths and circuits. (ADV)

G1.1.3 Use graph-coloring techniques to solve problems related to avoiding conflicts. (ADV)

G1.2 Lines and Angles

G1.2.1 Given a line and a point not on the line, construct (using compass and straightedge or dynamic geometry software) a line through the point that is parallel to the original line, and justify the steps of the construction.

G1.2.2 Given a line on the coordinate plane and a point not on the line, find an equation of the line through the point that is parallel to the given line.

G1.2.3 Given a line and a point not on the line, construct (using compass and straightedge or dynamic geometry software) a line through the point that is perpendicular to the original line, and justify the steps of the construction.

G1.2.4 Given a line and a point on the line, construct (using compass and straightedge or dynamic geometry software) a line through that point that is perpendicular to the given line, and justify the steps of the construction.

G1.2.5 Apply properties of vertical angles, linear pairs of angles, supplementary angles, complementary angles, and right angles to solve multi-step problems.

G1.2.6 Apply properties of corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles to solve multi-step problems.

G1.3 Triangles and Their Properties

G1.3.1 Solve multi-step problems involving angle measure, length, perimeter, or area of scalene, isosceles, equilateral, acute, right, or obtuse triangles.

G1.3.2 Know and use the Pythagorean Theorem, its converse, and Pythagorean Triples.

G1.3.3 Explain and apply the distance formula and midpoint formula in the plane.

G1.3.4 Know, justify, and use the properties of 30°- 60°- 90° triangles and 45°- 45°- 90° triangles.

G1.3.5 Know, justify, and use the properties of medians, altitudes, perpendicular bisectors of sides, and angle bisectors of a triangle, and construct these four special lines.

G1.3.6 Know and use the properties of the centroid, circumcenter, orthocenter, and incenter of triangles, and find these four special points using constructions.

G1.3.7 Describe symmetries of triangles.

G1.3.8 Recognize and explain how the rigidity and stability of triangles is utilized in real-world applications, such as trusses, tripods, tricycles, etc.

G1.3.9 Use the Hinge Theorem to explain how a triangular linkage (a triangle with a side that may vary in length) allows for controlled length and/or angle change, and yet preserves rigidity.

G1.3.10 Prove theorems about the angle sum of a triangle, and the exterior angles of a triangle.

G1.4 Quadrilaterals and Their Properties

G1.4.1 Solve multi-step problems involving angle measure, length, perimeter, or area involving squares, rectangles, parallelograms, kites, and trapezoids.

G1.4.2 Know, justify, and use properties of squares, rectangles, rhombi, parallelograms, kites, and trapezoids, and construct these quadrilaterals.

G1.4.3 Describe hierarchies among quadrilaterals, e.g. every rectangle is a parallelogram.

G1.4.4 Describe symmetries of quadrilaterals.

G1.4.5 Prove theorems about the interior and exterior angle sums of a quadrilateral.

G1.5 Other Polygons and Their Properties

G1.5.1 Dissect any polygon into non-overlapping triangles and use properties of triangles to analyze the polygon.

G1.5.2 Find the area of irregular polygonal regions using dissection.

G1.5.3 Identify regular n-gons from n=3 to n=10, and n=12.

G1.5.4 Know, justify, and use formulas for the perimeter and area of a regular n-gon and formulas to find interior and exterior angles of a regular n-gon.

G1.5.5 Describe symmetries of regular polygons.

G1.5.6 Explain why only certain polygons will tile the plane, and analyze and create regular and semi-regular tessellations using one or more polygons.

G1.6 Circles and Their Properties

G1.6.1 Solve multi-step problems involving circumference and area of circles.

G1.6.2 Explain how the area of a circle and the area of a parallelogram are related.

G1.6.3 Know and use properties of chords, tangent lines and tangent segments, and secant lines and secant segments of circles.

G1.6.4 Know and use properties of central angles, inscribed angles, and angles formed by intersecting chords in circles.

G1.6.5 Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.

G1.6.6 Know and use relations between circles and inscribed or circumscribed polygons.

G1.6.7 Recognize and explain how the rotational symmetry of a circle is key to many real-world applications.

G1.6.8 Apply concepts and properties of circles to analyzing gear ratios, pulleys, angular velocity, and linear velocity (ADV)

G1.7 Conic Sections and Their Properties

G1.7.1 Identify and distinguish among parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

G1.7.2 Identify real-world situations in which conic sections arise (e.g., the orbits of the planets are ellipses).

G1.7.3 Graph equations of parabolas and circles; write equations for parabolas and circles, given appropriate information.

G1.7.4 Graph equations for ellipses and hyperbolas; write equations for ellipses and hyperbolas. (ADV)

G1.7.5 Solve problems using properties of parabolas, circles, ellipses, and hyperbolas. (ADV)

G1.8 Coordinate Representations

G1.8.1 Use coordinates to describe points, lines, polygons, and circles in the plane.

G1.8.2 Use coordinate representations to justify properties of polygons and circles.

G1.8.3 Given a line on the coordinate plane and a point not on the line, find an equation of the line through the point that is perpendicular to the given line.

G1.8.4 Given a line on the coordinate plane and a point on the line, find an equation of the line through that point that is perpendicular to the given line.

G1.9 Polar Coordinates and Parametric Equations (ADV)

G1.9.1 Given polar coordinates, plot points in the plane. (ADV)

G1.9.2 Convert between polar and rectangular coordinate representation of points. (ADV)

G1.9.3 Graph and identify characteristics of polar equations, including lines, circles, cardioids, etc. (ADV)

G1.9.4 Convert between Cartesian and parametric form. (ADV)

G1.9.5 Graph equations in parametric form. (ADV)

STANDARD G2: THREE-DIMENSIONAL FIGURES AND THEIR PROPERTIES

Students represent and apply properties of polyhedra, round bodies, and 3-dimensional coordinate systems.

G2.1 Pyramids, Prisms, Polyhedra, and Their Properties

G2.1.1 Solve multi-step problems involving surface area and volume of pyramids and prisms.

G2.1.2 Describe symmetries of pyramids and prisms.

G2.1.3 Know and use Euler's formula relating the number of vertices, faces, and edges in polyhedra.

G2.1.4 Identify and describe the five Platonic solids (regular polyhedra): tetrahedron, cube, octahedron, dodecahedron, icosahedron, and recognize any of the five regular polyhedra given its net.

G2.1.5 Explain why there are exactly five regular polyhedra. (ADV)

G2.2 Cylinders, Cones, Hemispheres, and Spheres, and Their Properties

G2.2.1 Solve multi-step problems involving surface area and volume of cones, cylinders, hemispheres, and spheres.

G2.2.2 Describe symmetries of cones, cylinders, hemispheres, and spheres.

G2.3 Coordinate Representations

G2.3.1 Relate Global Positioning System (GPS) coordinates to the actual longitude, latitude, and altitude of a given location.

G2.3.2 Know and use the distance formula and midpoint formula in space. (ADV)

G2.3.3 Know and use the standard form of an equation for a plane. (ADV)

STANDARD G3: RELATIONS BETWEEN FIGURES

Students use and justify relations between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and justify statements about congruence, similarity, and self-similarity.

G3.1 Relations Between Area or Volume Formulas

G3.1.1 Explain the relation between the area formula of a triangle and the area formula of a parallelogram.

G3.1.2 Explain relations between the area formulas of various quadrilaterals, e.g., how to find the area of a trapezoid based on the area of a parallelogram or triangle.

G3.1.3 Explain the relation between the volume formulas for pyramids and prisms.

G3.1.4 Explain the relation between the volume formulas for cones and cylinders.

G3.1.5 Analyze the efficiency of the various 3-D forms by calculating the ratio of surface area to volume.

G3.2 Relations Between Two-dimensional and Three-dimensional Representations

G3.2.1 Given a 2-dimensional view, draw or construct the 3-dimensional object.

G3.2.2 Given a 3-dimensional figure, draw a 2-dimensional representation.

G3.2.3 Identify or draw cross-sections of 3-dimensional figures.

G3.2.4 Read and interpret a contour diagram, sketch the surface given a contour diagram, and draw a contour diagram given a data set.

G3.2.5 Given a figure and a line, draw or describe the solid that results from rotating the figure around the line. (ADV)

G3.3 Congruence and Similarity

G3.3.1 Prove that triangles are congruent by using the side-side-side (SSS), side-angle-side (SAS), angle-angle-side (AAS), angle-side-angle (ASA), and hypotenuse-leg (HL) theorems.

G3.3.2 Justify that the hypotenuse-angle, leg-leg, and leg-angle theorems in right triangles are special cases of the general triangle congruence theorems (SAS, ASA, SSS).

G3.3.3 Prove that other figures are congruent using properties of lines, segments, angles, triangles, or transformations, with and without use of coordinates.

G3.3.4 Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.

G3.3.5 Use properties of similar triangles and their corresponding parts to solve problems with and without use of coordinates.

G3.3.6 Know and use the Fundamental Theorem of Similarity: If a figure is scaled by a factor of k, corresponding lengths change by a factor of k, areas change by a factor of k^2, and volumes change by a factor of k^3.

STANDARD G4: TRANSFORMATIONS OF FIGURES IN THE PLANE

Students will solve problems about distance-preserving transformations, shape-preserving transformations and other transformations.

G4.1 Distance-preserving Transformations (Isometries)— Reflections, Rotations, Translations, Glide Reflections

G4.1.1 Find the image of a figure under a given isometry.

G4.1.2 Given two figures that are images of each other under an isometry, describe completely the isometry.

G4.1.3 Give a rule or mapping to describe a given isometry.

G4.2 Composite Transformations and Congruence

G4.2.1 Find the image of a figure under composition of two or more isometries, and tell whether the image is a reflection, rotation, translation, or glide reflection image of the preimage.

G4.2.2 Use transformations to create original figures that will tessellate (tile) the plane, and analyze tessellations of the plane (e.g., Escher's drawings or Islamic tilings).

G4.2.3 Given two congruent figures, tell which single transformation can be used to map one to the other.

G4.2.4 Use the Two Reflection Theorem to solve problems.

G4.2.5 Prove that figures are congruent using isometries. (ADV)

G4.3 Matrix Representations of Isometries

G4.3.1 Use matrices to represent and perform the following transformations of a point: reflection over the x-axis, y-axis, the lines $y = x$, or $y = -x$; rotation through an angle that is a multiple of 45°, and translations.

G4.3.2 Use matrices to find the image of a given polygon under transformations.

G4.4 Shape-preserving Transformations— Dilations (Size Changes) and Similarity Transformations

G4.4.1 Find the image of a figure under a dilation, given a center and size change factor using ruler and compass, coordinates, or dynamic geometry software.

G4.4.2 Given two figures that are images of each other under some dilation, identify the magnitude of the size change.

G4.4.3 Give a rule or mapping to describe a dilation with center at the origin and magnitude k.

G4.4.4 Find the image of a figure under a composite of a dilation and an isometry, with and without using coordinates.

G4.4.5 Given two similar figures, identify a dilation and an isometry that will map one to the other. (ADV)

G4.5 Other Transformations – Shears (Scale Changes)

G4.5.1 Find the image of a figure under a shear with center at the origin and horizontal scale factor h and vertical scale factor k.

G4.5.2 Given two figures that are images of each other under some shear, identify the center and scale factors of the shear.

G4.5.3 Give a rule or mapping to describe a shear with center at the origin and scale factors h (horizontal) and k (vertical).

G4.5.4 Explain how a shear can be used to find an equation for an ellipse centered at the origin and the area of the ellipse. (ADV)

STANDARD G5: TRIGONOMETRY AND VECTORS

Students solve problems about trigonometry, vectors, and the geometry of complex numbers.

G5.1 Triangle Trigonometry

G5.1.1 Express the sine, cosine, and tangent of angles in a right triangle.

G5.1.2 Use the sine, cosine, and tangent of an angle in a right triangle to solve problems.

G5.1.3 Describe how various strategies of measuring with triangles (e.g., similar triangles, Pythagorean Theorem, trigonometry) are used to find unknown distances and angles, and determine which strategy(ies) might be useful in a given situation.

G5.1.4 Solve problems using right triangle trigonometry, the Law of Sines, and the Law of Cosines.

G5.1.5 Find the area of a triangle using the formula $1/2\ a \cdot b \cdot \sin(\theta)$.

G5.2 Unit Circle Trigonometry

G5.2.1 Determine the sign of $\sin\theta$, $\cos\theta$, and $\tan\theta$, for $0 \le \theta \le 2\pi$, based on the unit circle.

G5.2.2 Convert between degrees and radians.

G5.2.3 Graph $y = \sin\theta$, $y = \cos\theta$, and $y = \tan\theta$ for $0° \le \theta \le 360°$ and $0 \le \theta \le 2\pi$.

G5.2.4 Describe the domain, range, and period of the sine, cosine, and tangent functions.

G5.2.5 Use basic trigonometric identities including $\sin^2\theta + \cos^2\theta = 1$, $\cos\theta = \sin(90 - \theta)$, and $\tan\theta = \sin\theta/\cos\theta$, to solve problems.

G5.2.6 Prove basic trigonometric identities (ADV).

G5.3 The Geometry of Complex Numbers and Vectors

G5.3.1 Interpret the results of the addition or subtraction of two vectors, and the multiplication of a vector by a scalar, geometrically.

G5.3.2 Solve problems about navigation using vectors.

G5.3.3 Relate the result of addition of complex numbers to translation and vectors, and the result of multiplication to rotation.

G5.3.4 Use DeMoivre's Theorem to find powers of complex numbers; describe the visual patterns that result when the powers are plotted in the complex plane. (ADV)

G5.3.5 Find the n^{th} roots of a complex number, represent them geometrically, and discuss visual patterns and symmetry. (ADV)

G5.3.6 Find the area of a triangle, given the coordinates of its vertices, using matrix methods. (ADV)

G5.3.7 Find the dot product and cross product of two vectors and interpret the result geometrically. (ADV)

G5.3.8 Use vectors to decompose force or motion into x- and y-components. (ADV)

STATISTICS AND PROBABILITY (S)

In Kindergarten through Grade 8, students develop the ability to read, analyze, and construct a repertoire of statistical graphs. Students also examine the fundamentals of experimental and theoretical probability. The Basic Counting Principle and tree diagrams serve as tools to solve simple counting problems.

During high school, students build on that basic foundation. They develop the data analysis and decision making skills that will serve them in their further study of mathematics as well as in their coursework in the physical, biological, and social sciences. These skills are also highly valuable outside of school, whether in their job or career or in day-to-day life. Students learn important skills in data collection, display, and analysis of both univariate and bivariate data. They utilize and analyze sampling methods, apply principles of effective data analysis and data presentation, and design and analyze experiments. Statistical process control is included as a widely used workplace technique. Inferential statistics is introduced, and advanced students are encouraged to work with confidence intervals and hypothesis testing. In probability, students utilize simulation strategies to estimate and probability models to calculate probabilities and make decisions. The normal distribution and its properties are studied, as are other key probability distributions. Students then use their understanding of probability to make decisions, solve problems, and determine whether or not statements made involving probabilities of events are reasonable. Students utilize technology when appropriate, including spreadsheets. This strong background in statistics and probability enables students to be savvy decision makers and smart information consumers and producers who have a full range of tools in order to make wise choices.

STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS

Students plot and analyze univariate data by considering shape of distributions and analyzing outliers; find and interpret commonly-used measures of center and variation; and explain and use properties of the normal distribution.

S1.1 Producing and Interpreting Plots

S1.1.1 Construct and interpret dot plots, histograms, relative frequency histograms, and box plots with appropriate labels and scales.

S1.1.2 Given a distribution, describe its shape, including symmetry or skewness, and state what effect the shape has on measures of center (mean and median) and measures of variation (range and standard deviation).

S1.1.3 Identify and give possible explanations for any outliers in a distribution; describe how they affect measures of center and variation.

S1.1.4 Recognize how linear transformations of univariate data affect shape, center, and spread (variation).

S1.2 Measures of Center

S1.2.1 Calculate and interpret measures of center including: arithmetic mean, median, mode, and weighted mean (weighted average); explain uses, advantages and disadvantages of each measure; determine which measure (or measures) are appropriate to use given a particular set of data and its context.

S1.2.2 Identify the position of the mean, median, and mode in both symmetrical and skewed distributions.

S1.2.3 Estimate the mean and median from a frequency distribution or histogram.

S1.3 Measures of Variation

S1.3.1 Compute and interpret measures of variation, including range, percentiles, quartiles, and interquartile range.

S1.3.2 Compute and interpret measures of variation including mean absolute deviation, variance, and standard deviation.

S1.4 The Normal Distribution

S1.4.1 Describe characteristics of the normal distribution, including its shape (bell shape, symmetric about the mean, the curve approaches the x-axis but never touches it or crosses it) and the relationship between its mean, median, and mode.

S1.4.2 State, explain, and apply the 68-95-99.7 Rule (Empirical Rule) for the normal distribution, regarding the percentage of observations lying within 1, 2, or 3 standard deviations of the mean.

S1.4.3 Calculate and interpret standardized scores (z-scores).

STANDARD S2: BIVARIATE DATA—EXAMINING RELATIONSHIPS

Students plot and analyze bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and finding and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.

S2.1 Producing and Interpreting Scatterplots

S2.1.1 Construct scatterplots with appropriate labels and scales.

S2.1.2 Describe shapes of scatterplots, identify clusters and different types of outliers.

S2.2 Linear Regression and Correlation

S2.2.1 Calculate and interpret Pearson's correlation coefficient for bivariate data that form a linear (elliptical) pattern; in particular, visually recognize perfect positive correlation, perfect negative correlation, no correlation, weak correlation, and strong correlation.

S2.2.2 Recognize how linear transformations of bivariate data affect the correlation.

S2.2.3 Differentiate between correlation and causation, and know that a strong correlation does not imply a cause-and-effect relationship.

S2.2.4 Define the concept of lurking variable and identify possible lurking variables in a given context.

S2.2.5 For bivariate data which appear to form a linear pattern, find the line of best fit, by estimating visually and by calculating the equation of the regression line; interpret the slope and y-intercept of regression equations.

S2.2.6 Use an estimated line of best fit and the equation of the regression line to make appropriate predictions.

S2.3 Linear and Nonlinear Models

S2.3.1 Determine whether a bivariate distribution forms a linear pattern, quadratic pattern, exponential pattern, logarithmic pattern, or none of these.

S2.3.2 Use technology to fit a regression model to bivariate data which appear to form a linear pattern, quadratic pattern, exponential pattern, or logarithmic pattern.

S2.3.3 Use a given nonlinear regression model to make appropriate predictions.

STANDARD S3: SAMPLE SURVEYS AND EXPERIMENTS

Students understand and apply sampling and various sampling methods, examine and design surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias, and understand and apply principles of good experiment design.

S3.1 Selecting Samples

S3.1.1 Distinguish between a survey and a census; describe advantages and disadvantages of each.

S3.1.2 Differentiate between a sample and a population, a statistic and a parameter.

S3.1.3 Explain how sample size affects the results of sample surveys, and determine appropriate sample sizes given desired confidence levels and maximum error levels.

S3.1.4 Explain various sampling methods and know their advantages and disadvantages; methods include simple random sampling, systematic random sampling, stratified random sampling, cluster sampling, probability sampling, voluntary response sampling, and convenience sampling.

S3.1.5 Identify possible sources of bias in data collection methods, including the impact of sampling methods and the phrasing of questions asked; describe how such bias can be reduced and controlled; and explain the impact of such bias on the conclusions that can rightly be made.

S3.1.6 Design surveys to collect data to answer questions of interest.

S3.2 Designing Experiments

S3.2.1 Distinguish between an observational study and an experiment, discuss advantages and disadvantages of each, and state what conclusions can be legitimately drawn from each.

S3.2.2 Describe principles of good experiment design, including the concepts of treatment group, control group, placebo effect, double-blind experiments, role of randomization in assigning individuals to groups, and replication.

S3.2.3 Design simple experiments involving two or more treatments to collect data to answer questions of interest.

STANDARD S4: STATISTICAL INFERENCE— DRAWING CONCLUSIONS FROM DATA

Students understand and use randomization tests to determine whether the result from an experiment is statistically significant; hypothesis testing and finding confidence intervals are included for advanced study. They understand basic elements of statistical process control.

S4.1 Informal Significance Testing

S4.1.1 Explain the meaning of "statistically significant" and "margin of error."

S4.1.2 Use a randomization (permutation) test to check if the results of a two-treatment experiment are statistically significant.

S4.2 Confidence Intervals

S4.2.1 Find and interpret large-sample confidence intervals for a proportion.(ADV)

S4.2.2 Find and interpret large-sample confidence intervals for a mean.(ADV)

S4.3 Hypothesis Tests (ADV)

S4.3.1 Describe the logic of hypothesis testing including ideas of null and alternative hypothesis; p-value; one and two-sided tests; and Type I and Type II errors. (ADV)

S4.3.2 Perform and interpret the results of a large-sample hypothesis test for a proportion. (ADV)

S4.3.3 Perform and interpret the results of a large-sample hypothesis test for a mean. (ADV)

S4.3.4 Perform and interpret the results of a large-sample hypothesis test for the difference between two proportions. (ADV)

S4.3.5 Perform and interpret the results of a large-sample hypothesis test for the difference between two means. (ADV)

S4.4 Statistical Process Control

S4.4.1 Recognize when the mean and standard deviation change on a plot over time.

S4.4.2 Use control charts and tests for out-of-control behavior.

S4.4.3 Explain why it is best to watch a process for awhile before trying to adjust it.

S4.4.4 Compute the probability of a false alarm on a set of readings, that is, the probability that a test will give an

out-of-control signal for a process that is under control.

STANDARD S5: STATISTICAL AND PROBABILISTIC REASONING

Students apply all of the data display and analysis tools they have learned as they critically examine graphs, charts, and reports; consider bias or misrepresentation; utilize statistical process control techniques, and prepare their own comprehensive data analysis presentations using graphics, written reports, and technology tools. They use basic elements of probabilistic reasoning.

S5.1 Comprehensive Data Analysis and Presentation

S5.1.1 Compare two or more sets of data using plots and summary statistics; make and justify conclusions or decisions.

S5.1.2 Interpret tables, charts, and plots of data and evaluate the appropriateness of the display for the data. (Includes all graph types from K-8 GLCE as well as more complex versions (multiple line graphs; multiple box plots; multiple bar graphs; artistic graphs such as those found in USA Today).

S5.1.3 Using data from tables, charts, and plots, answer questions which require multi-step calculations or comparisons.

S5.1.4 Given a table, plot, or report which has appeared in the media, analyze and critique it looking particularly for possible misrepresentation of the data, bias in the methods of data collection, the type of source and reliability of source, and whether the conclusions drawn by the media are supported by the data.

S5.1.5 Beginning with unorganized data and a question, organize and summarize the data, and prepare a written summary of findings.

S5.1.6 Use technology, spreadsheets in particular, to organize data, perform basic data analysis (measures of center, measures of variation, correlation coefficient – if any) and produce appropriate plots of the data.

S5.2 Probabilistic Reasoning

S5.2.1 Describe the classical (theoretical), the relative frequency (experimental), and the subjective approaches to probability; state advantages and disadvantages of each approach.

S5.2.2 Recognize and evaluate common probability misconceptions such as "hot streaks," "being due," "number of successes must approach expected number of successes."

STANDARD S6: SIMULATION AND THE LAW OF LARGE NUMBERS

Students apply a variety of simulation strategies as they extend their understanding of probability, including understanding and interpreting the Law of Large Numbers.

S6.1 Simulation

S6.1.1 Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation.

S6.1.2 Compare results of simulations to theoretical probabilities.

S6.2 Law of Large Numbers

S6.2.1 Know and interpret the Law of Large Numbers.

STANDARD S7: PROBABILITY MODELS AND CALCULATING PROBABILITIES

Students understand probability and find probabilities in more complex situations, including: compound events, using counting strategies, using geometric models, finding expected value, and utilizing diagrams and tables.

S7.1 Probability and Counting Strategies

S7.1.1 Determine the number of possible arrangements for situations where repetitions of items are permitted, and calculate probabilities of associated events. (ADV)

S7.1.2 Determine probabilities of events using permutations and combinations, tree diagrams, Venn diagrams, decision trees, and contingency (two-way) tables.

S7.2 Calculating Probabilities

S7.2.1 List sample spaces in situations where outcomes are equally likely and use them to find probabilities of events.

S7.2.2 Use geometric models when appropriate to find probabilities of events.

S7.2.3 Determine the odds in favor of or against an event given a list of outcomes or the event's probability; determine the probability of an event given the event's odds in favor of or against occurring.

S7.2.4 Calculate probabilities applying the addition (mutually exclusive events) and multiplication rules (independent events; dependent events), and the complement rule (complementary events).

S7.2.5 Find probabilities of conditional events.

S7.2.6 Compute binomial probabilities and solve applied problems. (ADV)

STANDARD S8: PROBABILITY DISTRIBUTIONS

Students understand and apply a repertoire of probability distributions, including the normal distribution. The Central Limit Theorem is included for advanced study.

S8.1 Probability Distributions: Uniform, Normal, Binomial, Geometric

S8.1.1 Describe features of, give common examples of, and interpret various probability distributions, including the uniform distribution, the normal distribution, the binomial distribution, and the geometric (waiting time) distribution.

S8.1.2 Determine the expected value of a discrete random variable.

S8.1.3 Use the area under the normal curve and z-scores to calculate the probability of an event.

S8.2 Central Limit Theorem

S8.2.1 Explain and apply the Central Limit Theorem. (ADV)

NOTES

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